UNIVERSITÉ DE STRASBOURG

Leading a continuation method by geometry for solving geometric constraints

SIAM Conference on
GEOMETRIC & PHYSICAL MODELING
(GD/SPM13)

November 11-14, 2013
the Curtis–a DoubleTree by Hilton Hotel
Denver, Colorado, USA

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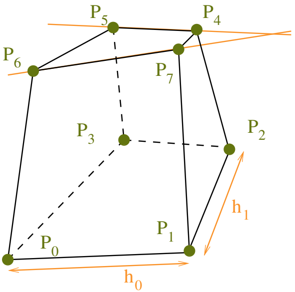
Geometric Constraint Systems (GCS)

- Set of geometric objects
  - points, lines, circles, ...
  - unknowns: $Point : P_0, P_1$, ...
- Satisfying a set of constraints
  - angles, distances, incidences, ...
  - terms: $h_0 = distance(P_0, P_1)$, ...
- Given *via* a sketch in CAD context



Unknowns:
$Point : P_0, P_1, \ldots, P_7$
Parameters:
$Length : h_0, h_1, \ldots$
Constraints:
$h_0 = distance(P_0, P_1)$
$h_1 = distance(P_0, P_2)$
$\ldots$
$coplanar(P_4, P_5, P_6, P_7)$
$\ldots$

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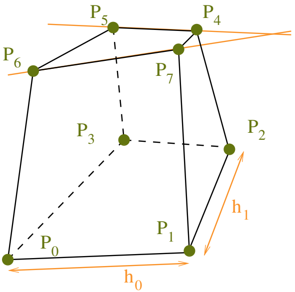

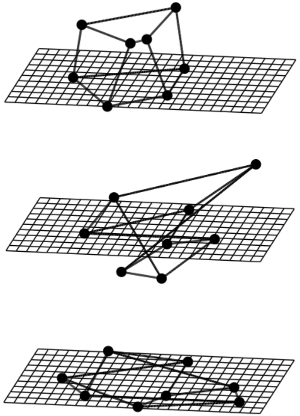

Structure of solution space

Constraints invariant by isometries: solutions sought modulo isometries

A GCS G is said generically

- well constrained : finite number of sol. (mod. isometries)
- under constrained : infinite number of sol. (mod. isometries)
- over constrained : no sol. (mod. isometries)

for almost all values of parameters (in an open set).

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Combinatorial Characterization: Laman Criterion

c: number of constraints, n: number of points
$G' < G$ with c': number of constraints, n': number of points

- $c = 2n - 3$
- $\forall G' < G$, $c' \leq 2n' - 3$

In 2D: $\Leftrightarrow$ gen. well constrainted points/distances GCS

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Combinatorial Characterization: Laman Criterion

c: number of constraints, n: number of points

$G' < G$ with c': number of constraints, n': number of points

- $c = 3n - 6$
- $\forall G' < G$, $c' \leq 3n' - 6$

In 3D: no sufficient condition is known.

Resolution

Symbolic resolution

- Locus Intersection Method (LIM), Knowledge based systems
- Algebraic approach

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$$F : \mathbb{R}^{18} \rightarrow \mathbb{R}^{18}$$

$$\begin{pmatrix} x_0 \\ \ldots \\ x_{17} \\ \\ \end{pmatrix} \mapsto \begin{pmatrix} \|\overrightarrow{P_0P_1}\| - h_0 \\ \|\overrightarrow{P_1P_2}\| - h_1 \\ \ldots \\ det(\overrightarrow{P_0P_1}, \overrightarrow{P_0P_2}, \overrightarrow{P_0P_3}) \\ \ldots \\ det(\overrightarrow{P_4P_5}, \overrightarrow{P_4P_6}, \overrightarrow{P_4P_7}) \end{pmatrix}$$

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$$F : \mathbb{R}^{18} \times \mathbb{R}^{12} \rightarrow \mathbb{R}^{18}$$

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"Hybrid" approaches

- Reparameterization
- Decomposition

Main contributions

Global homotopy approach

- homotopy method specialized to geometric constraint solving
- several solutions found, similar to the sketch
- deals with non-homogeneous solutions sets dimensions

Usage of constructive geometry

- finding other solutions

Some numerical results

Comparison with the free software `HOM4PS-2.0`:

- homotopy resolution of systems of polynomial equations
- finds all solutions

Table: Execution times[1].

	`HOM4PS-2.0`:	global homotopy:
Disulfide:		
nb solutions	18	8
time	6129s	3s
Hexahedron:		
nb solutions	16	7
time	12800s	1.6s
Icosahedron:		
nb solutions	-[2]	28
time	-	9s

[1] on an `Intel(R) Core(TM) i5 CPU 750 @ 2.67GHz`

[2] computation interrupted after a week

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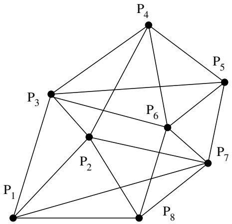

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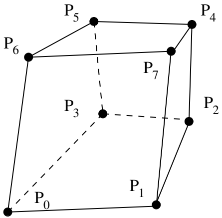

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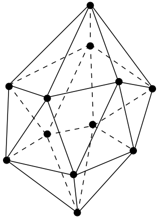

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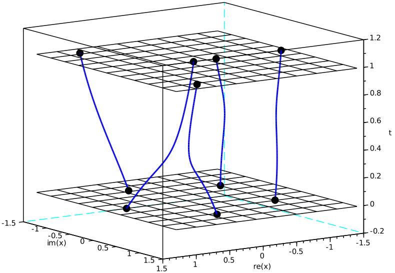
Homotopy methods

Main idea: Continuous deformation

- of an initial system $F_0(x) = 0$ (known solutions)
- into a target system $F(x) = 0$ (sought solutions)

via an homotopy function $H : \mathbb{C}^n \times [0,1] \to \mathbb{C}^n$

Example:
Target system:
$x^5 + 3x^2 + x = 0$
Initial system:
$\gamma\ (x^5 - 1) = 0, \gamma \in \mathbb{C}$
Homotopy function:
$H(x,t) = (1-t)F_0(x) + tF(x)$

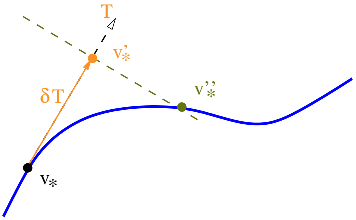


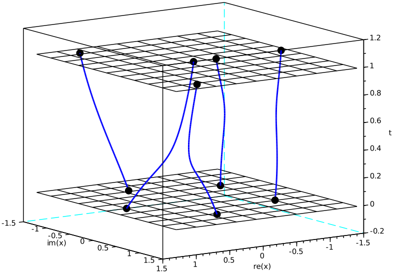
Homotopy methods

Main Result:

If H is regular, connected components of the set $\{(x,t) \in \mathbb{C}^n \times [0,1] | H(x,t) = 0\}$ are smooth manifolds of dimension 1: the homotopy paths.

Prediction-correction method:





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Example:

Target system:

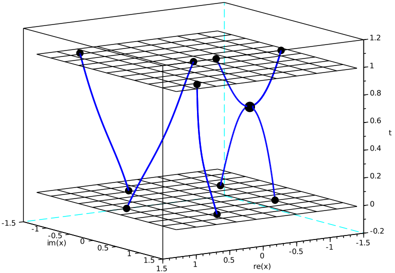
$$x^5 + 3x^2 + x = 0$$

Initial system:

$$(x^5 - 1) = 0$$

Homotopy function:

$$H(x,t) = (1-t)F_0(x) + tF(x)$$



Homotopy methods

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If H is regular, connected components of the set $\{(x,t) \in \mathbb{C}^n \times [0,1] | H(x,t) = 0\}$ are smooth manifolds of dimension 1: the homotopy paths.

Gamma trick: for almost all $\gamma \in \mathbb{C}^n$,

no critical points

H is regular

paths are strict. increasing with t

Example:

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$$x^5 + 3x^2 + x = 0$$

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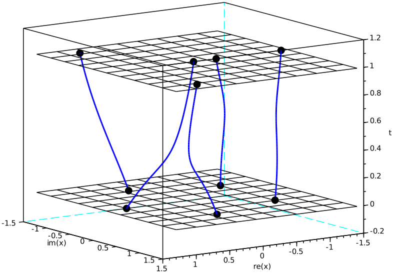
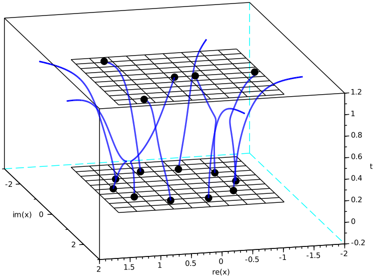


Homotopy methods

Over-estimation of zeros of F:

- several paths reach the same solution
- solutions at infinity (detected in $\mathbb{P}^n$)

Example:
Target system:
$$x^5 + 3x^2 + x = 0$$
Initial system:
$$\gamma(x^{10} - 1) = 0, \gamma \in \mathbb{C}$$
Homotopy function:
$$H(x,t) = (1-t)F_0(x) + tF(x)$$



Application: Classical homotopy method

Building homotopy function:

Parameters:

a_{so} given by the user

Target system: numerical function $F(x, a_{so})$

Advantage:

- all isolated solutions are found

Disadvantages: high computational cost

- number of zero of F over-estimated
- most of solutions are complex

[DH00] C. Durand and C.M. Hoffmann.
A systematic framework for solving geometric constraints analytically.
Journal of Symbolic Computation, 30(5):493–519, 2000.

Application: Using sketch

Building homotopy function: Sketch: $x_{sk} \in \mathbb{R}^n$
Parameters: a_{sk} read on the sketch
a_{so} given by the user
Homotopy function: $F(x, (1-t)a_{sk} + ta_{so})$

Advantage: low computational cost

- a single path is followed
- solution similar to the sketch

Disadvantages:

- a single solution is found
- path is followed $\mathbb{R}^n \times [0, 1]$

[LM95] Hervé Lamure and Dominique Michelucci.
Solving geometric constraints by homotopy.
pages 263–269, 1995.

Considering homotopy path in $\mathbb{R}^n \times [0,1]$

$(x,t) \in \mathcal{S}$, $\mathcal{S} \in \mathbb{R}^n \times [0,1]$ homotopy path of H

- solution for parameters $(1-t)a_{sk} + ta_{so}$
- only real solutions are found

Unknowns:
Point $P_0, \dots, P_5$
Parameters:
Length $h_0, \dots, h_{11}$
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$\dots$
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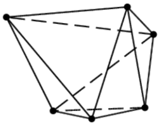

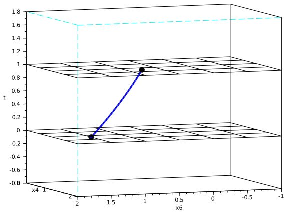

Considering homotopy path in $\mathbb{R}^n \times \mathbb{R}$

$(x, t) \in \mathcal{S}$, $\mathcal{S} \in \mathbb{R}^n \times \mathbb{R}$ homotopy path of H

- solution for parameters $(1 - t)a_{sk} + ta_{so}$
- only real solutions are found
- several (sometimes all) solutions are found

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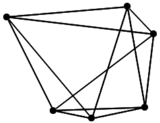

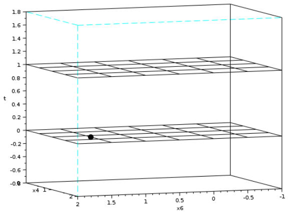

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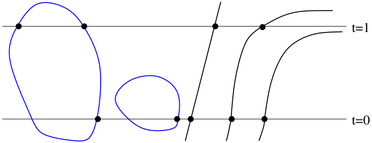
If H is regular its homotopy paths in $\mathbb{R}^n \times \mathbb{R}$ are diffeomorphic

- to a circle
  $\Rightarrow$ several solutions
  $\Rightarrow$ termination criterion
- to a line
  $\Rightarrow$ infinite computation



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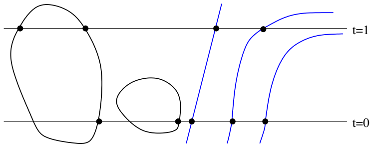

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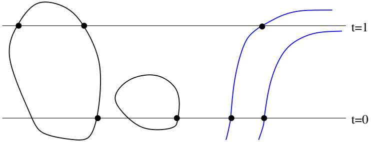
  $\Rightarrow$ that lead to a solution at infinity
  its limit is a specific geometric configuration that can be detected



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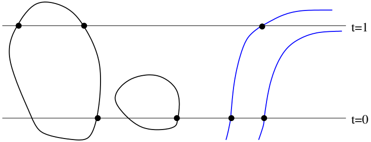
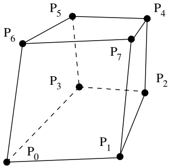


Example: the hexahedron problem

Problem 1: *Build an hexahedron, knowing length of its* 12 *edges.*

For allmost all values of parameters, its solutions are:

- isolated points



GCS 1:

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8 points	18 dof
Constraints:	
12 distances 6 coplanarity	18 dor

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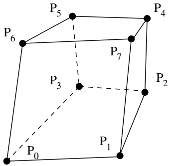
- solutions of problem 2

Problem 2: *Build* 8 *points in a plane, knowing* 12 *distances pairwise.*



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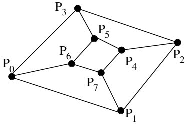

GCS 2:

Unknowns:	
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Constraints:	
12 distances	17 dor
5 coplanarity	

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- one dimensional manifolds

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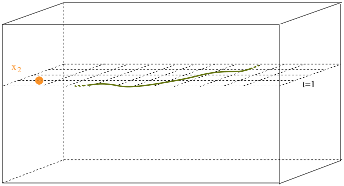

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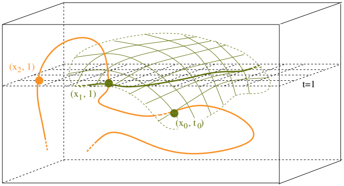
Homotopy function $H(X,t)$: For allmost all values of parameters, $\{(X,t)|H(X,t)=0\}$ admits critical points.



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Homotopy function $H(X,t)$: For allmost all values of parameters, $\{(X,t)|H(X,t)=0\}$ admits critical points.

Result:

Let $F(X,A)$ be the numerical function of a well constrained GCS. If $\forall i$, f_i depends of a parameter $a_i \in \mathbb{R}$, $F(X,(1-t)a_{sk}+ta_{so})$ is regular for allmost all $a_{sk} \in \mathbb{R}^n$, $a_{so} \in \mathbb{R}^n$.

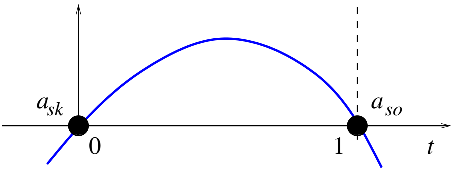
[LW93] TY Li and Xiao Shen Wang.
Solving real polynomial systems with real homotopies.
mathematics of computation, 60(202):669–680, 1993.

Parameterization of boolean constraints

Idea: *Substitute boolean constraints by constraints of parameters not nul when $t \notin [\![0,1]\!]$ to apply previous result.*

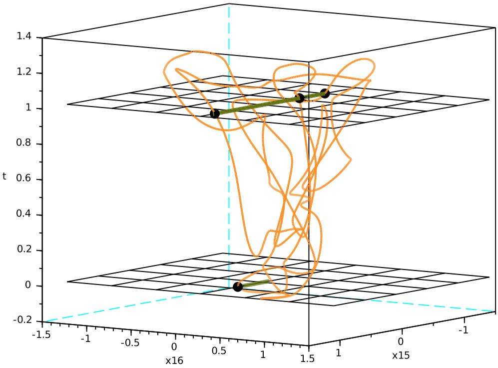
In practice:

$$det(\overrightarrow{P_0P_1}, \overrightarrow{P_0P_2}, \overrightarrow{P_0P_3}) \rightarrow det(\overrightarrow{P_0P_1}, \overrightarrow{P_0P_2}, \overrightarrow{P_0P_3}) - a$$



Heuristic result: For allmost all such interpolation functions C^∞, H does not admit critical points on $\mathbb{R}^n \times (\mathbb{R} \setminus [\![0,1]\!])$.

Example: the hexahedron problem



Main contributions

Global homotopy approach

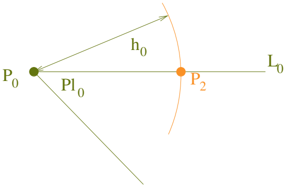
- homotopy method specialized to geometric constraint solving
- several solutions found, similar to the sketch
- deals with non-homogeneous solutions sets dimensions

Usage of constructive geometry

- finding other solutions

Construction plan

- Constructs figures (geometric objects)
  - by intersecting geometric locii
  - knowing parameters values



Paramètres:
P_0 , Pl_0 , L_0, h_0 , h_1, h_2 , . . . , h_9, h_{10}, h_{11}
Inconnues:
S_0 , P_2 , S_1, S_2 , P_1 , . . . , S_9, S_{10}, S_{11}, P_5
Termes: $S_0 = sphere(P_0, h_0)$
$P_2 = interPLS(Pl_0, L_0, S_0)$
$S_1 = sphere(P_0, h_1)$
$S_2 = sphere(P_2, h_2)$
$P_1 = interPSS(Pl_0, S_1, S_2)$
. . .
$S_9 = sphere(P_0, h_8)$
$S_{10} = sphere(P_1, h_9)$
$S_{11} = sphere(P_4, h_{10})$
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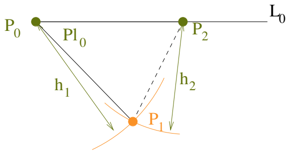

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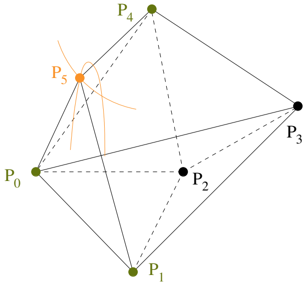

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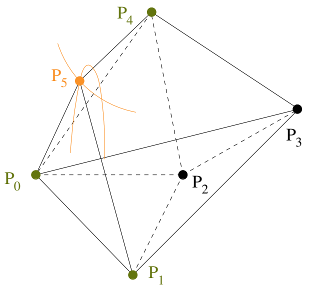
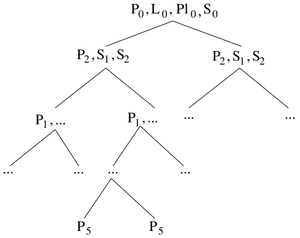
Construction plan

- Constructs figures (geometric objects)
  - by intersecting geometric locii
  - knowing parameters values
  - making choices





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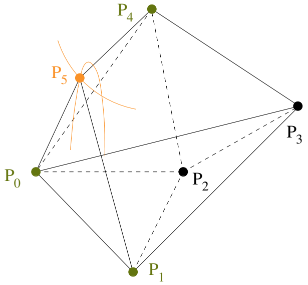

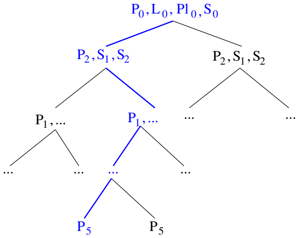

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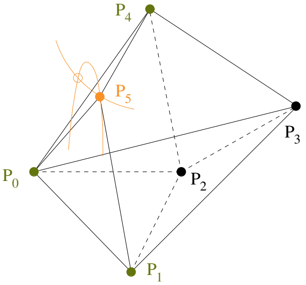
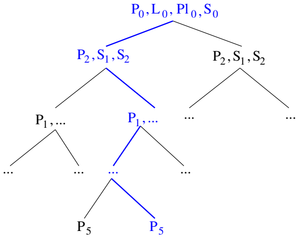
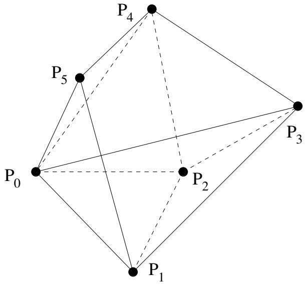




Construction plan

- Constructs figures (geometric objects)
- Obtained by
  - the Locus Intersection Method (LIM)



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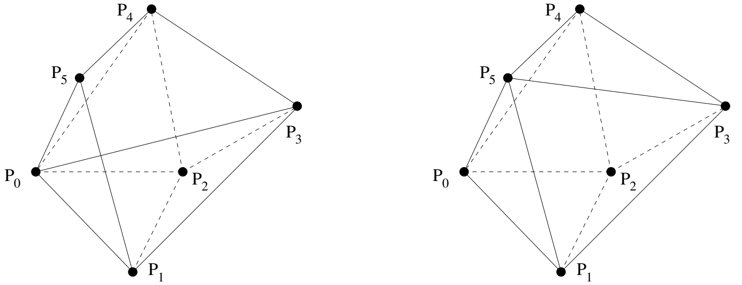
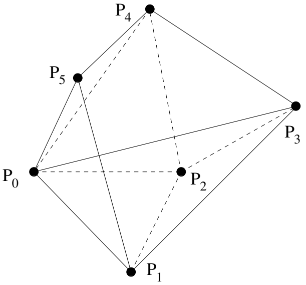
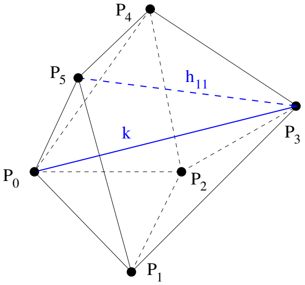


Construction plan

- Constructs figures (geometric objects)
- Obtained by
    - the Locus Intersection Method (LIM)
    - or reparameterization





Construction plan

- Constructs figures (geometric objects)
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Idea: Using a construction plan obtained by reparameterization to build new sketches.

Some numerical results

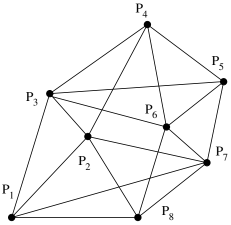
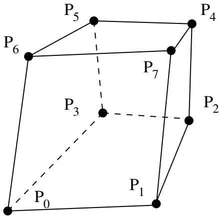


Table: Execution times[3].

	HOM4PS-2.0:	global homotopy:	Exhaustive Research:
Disulfide:			
nb solutions	18	9	13
time	6129s	0.4s	108s
Hexahedron:			
nb solutions	16	7 (3)	16
time	12800s	1.6s	136s
Icosahedron:			
nb solutions	-[4]	28	308[5]
time	-	9s	-

[3] on an Intel(R) Core(TM) i5 CPU 750 @ 2.67GHz
[4] computation interrupted after a week
[5] computation interrupted after 8 hours

Some numerical results



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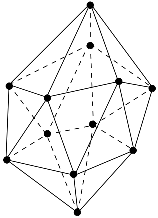

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Idea: semi-automatic research of new solutions

[3] on an `Intel(R) Core(TM) i5 CPU 750 @ 2.67GHz`

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Conclusion: homotopy method led by geometry to solve GCS:

- the sketch is used to determine the initial system
- several solutions are found
- a construction plan obtained by reparameterization constructs new sketches

Prospects:

- characterize the geometry of homotopy paths
- detecting geometric causes of heterogeneous solution sets dimension
- semi-automatic research of new solutions

Thank you for your attention

Collaboration opportunities are welcome.

`rimbach@unista.fr`